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HEAT EXCHANGE IN A TUBE FILLED WITH GRANULAR BED

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Results of the modeling of the process of heat transfer from a circular tube filled with granular bed with boundary conditions of the first, second, and third kind have been presented. The physical characteristics of the wall zone and the relative value of its thermal resistance have been determined based on an analysis of experimental data on the nonstationary heat exchange of the unblown granular bed. Recommendations on calculation of the heat exchange at elevated temperatures have been given.

Processes in which one has to supply or remove heat from the gas (fluid) flowing in a granular-bed-filled tube are frequently used in technology. Characteristic examples are provided by catalytic reactors for catalytic reactions and units for thermal processing of a solid fuel. To calculate the temperature distribution in the bed and the surface required for removal of a prescribed quantity of heat one must know the regularities of heat transfer in such a system. As is well known [1], the main problem arising in this case is correct account of the influence of the thermal resistance of the wall zone on the heat transfer from a wall of the tube filled with the granular medium. Experimental investigation of this question has been the objective of a fairly ample amount of literature (see, for example, [1–3]). Aérov et al. [1] have recommended for engineering calculations the following dependence:

$$\mathrm{Nu}_{\mathrm{w,e}} = \frac{2}{3}\frac{\lambda_{\mathrm{s}}^{0}}{\lambda_{\mathrm{f}}} + 0.09\mathrm{Re}_{\mathrm{e}}^{0.8}\,\mathrm{Pr}^{0.33}\,, \tag{1}$$

where $\lambda_s^0/\lambda_f = 5$ (non-heat-conducting particles) and $\lambda_s^0/\lambda_f = 15$ ((metallic) heat-conducting particles). The range of check of (1) is $\mathrm{Re_e} = 1–10^4$.

A more detailed experimental study of the process in filtration through a layer of water and a 47% glycerin solution, carried out recently in [3], has shown that the regularities of heat exchange in such a system are more complex than those described by formula (1). Experimental points in the coordinates $\mathrm{Nu_w/Pr^{0.4}}$, Re were markedly stratified in the region of inertial and transient flow regimes ($\mathrm{Re} < 120$), without enabling one to reveal a unified law of heat exchange. It is only in the turbulent regime ($\mathrm{Re} > 120$) that such a "universal" law has been established:

$$\mathrm{Nu_w} = 0.4\,\mathrm{Re}^{0.66}\,\mathrm{Pr}^{0.4}\,. \tag{2}$$

It is clear from what has been said above that our knowledge of the process of heat exchange in the system is still far from being adequate. As the analysis shows, this is primarily true of the physical characteristics of the wall zone and the influence of the type of boundary conditions and of the radiant heat transfer on the heat exchange.

1. Influence of the Type of Boundary Conditions. *1.1. Boundary Condition of the First Kind on the Exterior Tube Surface.* The system of equations for the two-band model has the form

$$c_{\mathrm{f}}\,\rho_{\mathrm{f}}u\,\frac{\partial T}{\partial x} = \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right);$$

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$$r = 0\,,\;\; \frac{\partial T}{\partial r} = 0\,;\;\;\; x = 0\,,\;\;\; T = T^0\,;$$
$$r = R - l_0\,,\;\; -\lambda\frac{\partial T}{\partial r} = K\,(T - T_0)\,, \tag{3}$$

where $K = 1/(1/\alpha_w + \delta/\lambda_m)$ is the heat-transfer coefficient allowing for the thermal resistance of the wall zone ($1/\alpha_w$) and the tube wall (δ/λ_m). To solve system (3) we write it in a dimensionless form coincident with the known problem of calculation of the nonstationary temperature field in an unbounded cylinder with boundary conditions of the third kind:

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{\partial^2\theta}{\partial\,(r')^2} + \frac{1}{r'}\frac{\partial\theta}{\partial r'}\,;$$
$$r' = 0\,,\;\; \frac{\partial\theta}{\partial r'} = 0\,;\;\;\; \mathrm{Fo} = 0\,;\;\;\; \theta = 1\,;$$
$$r' = 1 - l_0/R\,,\;\; -\frac{\partial\theta}{\partial r'} = \mathrm{Bi}\,\theta\,. \tag{4}$$

The solution of (4) has the form [4]

$$\theta = \sum_{n=1}^{\infty} A_n J_0\,(\mu_n r')\exp\,(-\,\mu_n^2\,\mathrm{Fo})\,, \tag{5}$$

where $A_n = \dfrac{2J_1(\mu_n)}{\{\mu_n[J_0^2(\mu_n) + J_1^2(\mu_n)]\}}$ and μ_n are the roots of the characteristic equation

$$\frac{J_0\,(\mu)}{J_1\,(\mu)} = \frac{1}{\mathrm{Bi}}\,\mu\,. \tag{6}$$

The series in (5) converges rapidly. When $\mathrm{Fo} > 0.1$ (this corresponds to $x > 0.1 c_f \rho_f u R^2/\lambda$, we can confine ourselves just to the first term of the series

$$\theta \approx A_1 J_0\,(\mu_1 r')\exp\,(-\,\mu_1^2\,\mathrm{Fo})\,. \tag{7}$$

For the heat-transfer coefficient determined by the relation

$$K_\Sigma = K\,\frac{T\big|_{r=R-l_0} - T_0}{\langle T\rangle - T_0}\,, \tag{8}$$

from (7) with account for $l_0 << R$ we have

$$K_\Sigma = K\,\frac{J_0\,(\mu_1)\,\mu_1}{2J_1\,(\mu_1)}\,. \tag{9}$$

We note that, in obtaining (9), we have employed the relation [4, p. 121]

$$\int_0^1 J_0(\mu_n r')\, r' dr' = \frac{1}{\mu_n} J_1(\mu_n) \,. \tag{10}$$

With account for (6), Eq. (9) yields

$$K_\Sigma = K \frac{\mu_1^2}{2\,\mathrm{Bi}} \,. \tag{11}$$

For μ_1 we have carried out the simple approximation

$$\mu_1 = \sqrt{\frac{2\,\mathrm{Bi}}{1 + \mathrm{Bi}/2.8915}} \,, \tag{12}$$

which enables us to obtain, from (11), the final expression for K_Σ:

$$K_\Sigma = 1 \Big/ \left(\frac{1}{\alpha_\mathrm{w}} + \frac{D}{5.78\lambda} + \frac{\delta}{\lambda_\mathrm{m}} \right). \tag{13}$$

Formula (13) involves all the components of the total thermal resistance.

1.2. Boundary Condition of the Third Kind on the Exterior Tube Surface. In the case of the heat exchange of the ambient medium with the exterior tube surface according to the Newton law with a heat-exchange coefficient α_w^* we can easily obtain the expression for the heat-transfer coefficient from the generalization of (13):

$$K_\Sigma^* = 1 \Big/ \left(\frac{1}{\alpha_\mathrm{w}} + \frac{D}{5.78\lambda} + \frac{\delta}{\lambda_\mathrm{m}} + \frac{1}{\alpha_\mathrm{w}^*} \right). \tag{13à}$$

1.3. Boundary Condition of the First Kind on the Inside of the Tube. The expression for the heat-transfer coefficient for this case immediately follows from (13) when $\delta/\lambda_\mathrm{m} \to 0$:

$$\alpha = 1 \Big/ \left(\frac{1}{\alpha_\mathrm{w}} + \frac{D}{5.78\lambda} \right). \tag{14}$$

1.4. Boundary Condition of the Second Kind. In this case, the system of equations will have the form

$$\begin{gathered} c_\mathrm{f} \rho_\mathrm{f} u \frac{\partial T}{\partial x} = \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right); \\ r = 0 \,, \quad \frac{\partial T}{\partial r} = 0 \,; \\ x = 0 \,, \quad T = T^0 \,; \\ r = R - l_0 \,, \quad -\lambda \frac{\partial T}{\partial r} = q \,. \end{gathered} \tag{15}$$

For the heat-exchange coefficient determined by the relation

$$\alpha = \alpha_\mathrm{w} \frac{T\big|_{r=R-l_0} - T_\mathrm{w}}{\langle T \rangle - T_\mathrm{w}} \,, \tag{16}$$

by solution of system (15) in [2] it has been obtained that

$$\alpha = 1 \Big/ \left(\frac{1}{\alpha_w} + \frac{D}{8\lambda} \right). \tag{17}$$

As is seen, formula (17) is similar to (14) in structure, and it differs only in the numerical coefficient in the expression of the thermal resistance of the granular bed. Combining (14) and (17), we obtain a generalized formula for the coefficient of heat exchange of the granular bed with the interior surface of a circular tube:

$$\alpha = 1 \Big/ \left(\frac{1}{\alpha_w} + \frac{D}{C\lambda} \right), \tag{18}$$

where C = 5.78 (boundary conditions of the first kind) and C = 8 (boundary conditions of the second kind).

As is seen, all the formulas obtained for the coefficients of heat transfer and heat exchange (13), (13a), (14), and (17) involve only one unknown parameter — the wall heat-exchange coefficient — which calls for its detailed physical analysis and determination.

2. Wall Coefficient of Heat Exchange. Taking into account the relative smallness of the wall-zone thickness, we can represent the quantity α_w in the form

$$\alpha_w = \lambda_{eff} / l_0 \, . \tag{19}$$

To determine l_0 we employed the results of experiments on nonstationary heat exchange of the granular bed of non-heat-conducting particles with the wall in the case of an immobile gas phase [5]. The processing of these data carried out in [6] in the coordinates Nu, Fo^* has led to the following result, which is important in the context of the present work:

$$\lim_{Fo^* \to 0} Nu = 10. \tag{20}$$

Taking into account the fact that the overall thermal resistance is concentrated in the wall region at short times, we can justifiably set

$$\lim_{Fo^* \to 0} Nu = Nu_w = 10. \tag{21}$$

Disregarding the influence of heat transfer in the spots of contact between the non-heat-conducting particles and the surface, we can set $\lambda_{eff} \approx \lambda_f$ for the case of the absence of blowing of the bed. Then, substituting the expression of α_w from (19) into (21), we have

$$l_0 = 0.1d \, . \tag{22}$$

We note that virtually the same result is obtained after the geometric averaging of variable thicknesses of gas lenses which are formed by spherically shaped particles adjacent to the tube wall [7].

For determination of λ_{eff} in the general case we assume the validity of the relation

$$\lambda_{eff} = A\lambda_f + Bc_f \, \rho_f ud \, , \tag{23}$$

which is analogous in form to the well-known dependence for calculation of the effective thermal conductivity of a granular bed with a moving gas phase [8]

$$\lambda = \lambda_s^0 + 0.1c_f \, \rho_f ud \, , \tag{24}$$

where the value of λ_s^0 is given by the expression

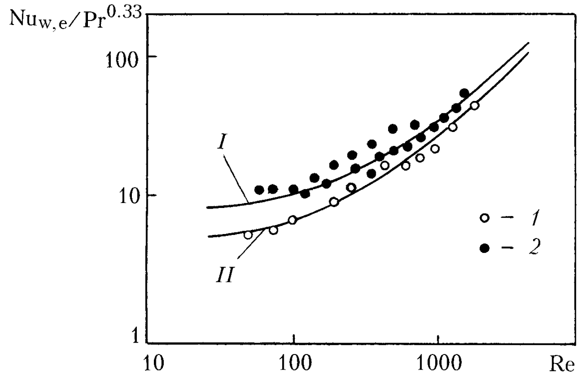


Fig. 1. Wall coefficient of heat exchange: I) calculation from (29) for A = 1.6; II) the same for A = 1; 1) data [10] for glass and silica-gel spheres; 2) the same for steel and lead spheres.

$$\frac{\lambda_s^0}{\lambda_f} = 1 + \frac{(1-\varepsilon)\,(1-\lambda_f/\lambda_s)}{\lambda_f/\lambda_s + 0.28\varepsilon^{0.63(\lambda_s/\lambda_f)^{0.18}}} . \tag{25}$$

As has been noted earlier, we have $\lambda_{eff} \approx \lambda_f$ for non-heat-conducting particles when u = 0. Consequently, here A = 1. In the case of heat-conducting (metallic) particles it is obvious that $A > 1$ with allowance for the influence of contact heat conduction. The specific value of A can also be determined by analysis of the experiments on nonstationary heat exchange of the granular beds of copper spheres [9]. In [6], it has been established that

$$\lim_{Fo^* \to 0} \mathrm{Nu} = \mathrm{Nu}_w = 16. \tag{26}$$

Substituting the expression of α_w from (19) into (26) for l_0 = 0.1d, we have the relation

$$\lambda_{eff} = 1.6\lambda_f , \tag{27}$$

which determines the value A = 1.6 in the case of heat-conducting particles. The coefficient B on the convective side of (23), which has remained unknown, can easily be found from a comparison of the calculated α_w values obtained from (19)

$$\alpha_w = \frac{10}{d}\,(A\lambda_f + Bc_f\rho_f ud) \tag{28}$$

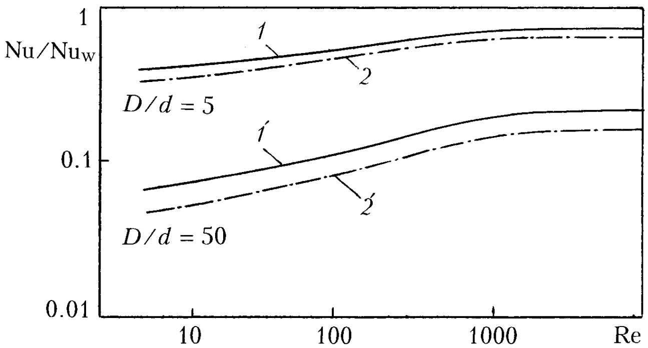
and the available experimental data. The employment of the results of determination of α_w in [10] has yielded $B \approx 0.0061$. It is of interest to note that $B \approx 0.0061/\varepsilon$ was obtained earlier in [11] for the case of a developed fluidized bed. The coefficient A was virtually equal to unity for both non-heat-conducting and conducting particles, which is, apparently, attributed to the high mobility of the particles at the heat-exchange surface. Finally, for calculation of the wall coefficient of heat exchange we have

$$\mathrm{Nu}_w = 10\,(A + 0.0061\,\mathrm{Re}\,\mathrm{Pr}) . \tag{29}$$

Figure 1 compares the values of α_w calculated from (29) and the experimental values of α_w in the case of heat-conducting and non-heat-conducting particles.

For the ratio of the thermal resistance of the wall zone to the total thermal resistance, Eq. (18) yields

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_w} = \frac{1}{1 + \dfrac{2\,\mathrm{Bi}^*}{C}} . \tag{30}$$



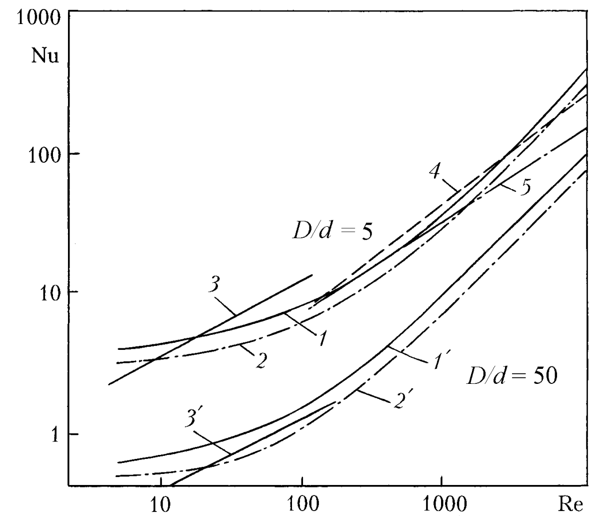


Fig. 2. Fraction of thermal resistance of the wall zone in the total thermal resistance for A = 1: 1 and 1′) calculation from (31) for C = 8; 2 and 2′) the same for C = 5.78.

Fig. 3. Coefficient of heat exchange in a circular tube filled with granular bed for A = 1: 1 and 1′) calculation from (32) for C = 8; 2 and 2′) the same for C = 5.78; 3 and 3′) calculation from (33); 4) calculation from (1); 5) calculation from (2).

With account for (24) and (29) we have

$$\frac{\mathrm{Nu}}{\mathrm{Nu_w}} = \frac{1}{1 + \dfrac{D}{dC}\,\dfrac{10A + 0.061\,\mathrm{Re\,Pr}}{\lambda_s^0/\lambda_f + 0.1\,\mathrm{Re\,Pr}}}\,. \tag{31}$$

Figure 2 shows the calculation of $\mathrm{Nu/Nu_w}$ for the following parameters: A = 1, λ_s^0/λ_f = 4, and Pr = 0.72. Figure 3 shows the calculation of the heat-exchange coefficient for the same values of the parameters according to the formula

$$\mathrm{Nu} = \frac{1}{\dfrac{1}{10A + 0.061\,\mathrm{Re\,Pr}} + \dfrac{D/d}{C\,(\lambda_s^0/\lambda_f + 0.1\,\mathrm{Re\,Pr})}}\,, \tag{32}$$

obtained from (18), (24), and (28). The same figure shows dependences (1) and (2) for the wall coefficient of heat exchange and the equation established in [3] for the inertial regime of heat exchange ($\mathrm{Re} < 120$):

$$\mathrm{Nu} = 7.5\,\frac{d}{D}\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.33}\,. \tag{33}$$

As is clear from Fig. 3, when the Re numbers are higher than average, the values of Nu and $\mathrm{Nu_w}$ are fairly close for small D/d numbers. This indicates the dominant role of the thermal resistance of the wall zone under these conditions.

Based on formulas (31) and (32) we can easily obtain the limiting values of the quantities $\mathrm{Nu/Nu_w}$ and Nu for large Re:

$$\lim \frac{\mathrm{Nu}}{\mathrm{Nu_w}} = \frac{1}{1 + \dfrac{0.61D}{Cd}}\,, \tag{34}$$

$$\lim \mathrm{Nu} = \left(\frac{1}{16.4 + \dfrac{10D}{Cd}} \right) \mathrm{Re}\,\mathrm{Pr}\,. \tag{35}$$

3. Influence of Radiant Transfer. It is easy to generalize the results obtained to the case of elevated temperatures where the radiant transfer of heat becomes substantial. For this purpose it is proposed that the component of the effective thermal conductivity of the granular bed λ_s^0 in formula (24) be calculated according to the Kunii model [1, pp. 104–105] allowing for the influence of the radiant transfer of heat on the quantity λ_s^0:

$$\frac{\lambda_s^0}{\lambda_f} = \varepsilon \left(1 + \frac{\alpha_1 d}{\lambda_f} \right) + \frac{1 - \varepsilon}{\dfrac{1}{\dfrac{1}{\phi} + \dfrac{\alpha_2 d}{\lambda_f}} + \dfrac{2\lambda_f}{\lambda_s}}\,, \tag{36}$$

where

$$\alpha_1 = 0.227 \left[\frac{1}{1 + \varepsilon\,(1 - p)/(2p\,(1 - \varepsilon))} \right] \left(\frac{T}{100} \right)^3 ; \tag{37}$$

$$\alpha_2 = 0.227\,\frac{p}{2 - p} \left(\frac{T}{100} \right)^3 . \tag{38}$$

For the parameter φ, when $\varepsilon \approx 0.4$, we have proposed the following simple approximation:

$$\phi = 0.25 \left(\frac{\lambda_s}{\lambda_f} \right)^{-0.27} . \tag{39}$$

Rough calculations show that $\lambda_s^0/\lambda_f \approx 8$ (room temperatures), $\lambda_s^0/\lambda_f \approx 16$ (T = 873 K), and $\lambda_s^0/\lambda_f \approx 25$ (T = 1073 K) for ε = 0.4 and $\lambda_s^0/\lambda_f \approx 100$. In accordance with this, the heat-exchange coefficient markedly increases (especially for large D/d). Thus, in accordance with (32), we have $\mathrm{Nu}\big|_{T=1073}/\mathrm{Nu}\big|_{T=273} \approx 2$ (for D/d = 50, A = 1, C = 8, Re = 100, and Pr = 0.72).

Conclusions. As a result of the investigation carried out, we have determined the most important physical characteristics of the wall zone of a granular bed — the zone's thickness (22) and effective thermal conductivity (23), where A = 1 (non-heat-conducting particles), A = 1.6 (heat-conducting particles), and B = 0.0061. We have obtained the expression for calculation of the wall coefficient of heat exchange (29) and the general dependence for the coefficient of heat exchange of the granular bed with the interior surface of a circular tube, which allows for the influence of the type of boundary conditions (32). Within the framework of this dependence, we have carried out a generalization to the case of elevated temperatures where radiant heat transfer becomes substantial.

NOTATION

a_f, thermal diffusivity of the gas (fluid), m^2/sec; A and B, coefficients determined in (23); Bi = KR/λ and Bi* = $\alpha_w R/\lambda$, Biot numbers; C, coefficient determined in (18); c_f, heat capacity of the gas (fluid), J/(kg·deg); d, diameter of particles, m; $d_e = 4\varepsilon d/6(1 - \varepsilon)$, equivalent (hydraulic) diameter of particles, m; D, inside diameter of the tube, m; Fo = $\lambda x/\rho_f c_f u R^2$ and Fo* = $a_f t/d^2$, Fourier numbers; J_0 and J_1, Bessel functions of the first kind of zero and first order respectively; l_0, thickness of the wall zone, m; Nu = $\alpha d/\lambda_f$, Nu$_w$ = $\alpha_w d/\lambda_f$, and Nu$_{w,e}$ = $\alpha_w d_e/\lambda_f$, Nusselt numbers; p, emittivity of the particle surface; Pr = $\mu_f c_f/\alpha_f$, Prandtl number; q, heat flux, J/(m^2·sec); r, radial coordinate, m; $r' = r/R$; $R = D/2$, m; Re = $ud\rho_f/\mu_f$ and Re$_e$ = $ud_e\rho_f/\varepsilon\mu_f$, Reynolds numbers; t, time, sec; T, absolute temperature, K; $\langle T \rangle$, temperature average over the cross section x = const, K; T^0, temperature of the gas (fluid) at the inlet to the tube, K; T_0, ambient temperature, K; u, rate of filtration of the gas (fluid), m/sec; x, longitudinal coordinate, m; α, heat-exchange coefficient, W/(m^2·deg); δ, thickness of the tube wall, m; ε, porosity; $\theta = (T - T_0)/(T^0 - T_0)$, dimensionless relative temperature; λ, effective thermal conductivity of the granular bed, W/(m·deg); λ_m, thermal conductivity of the tube material, W/(m·deg); λ_s^0, thermal conductivity of the granular bed for $u = 0$, W/(m·deg); λ_s, thermal conductivity of the particle material, W/(m·deg); λ_f, thermal conductivity of the gas (fluid), W/(m·deg); λ_{eff}, effective thermal conductivity of the wall zone, W/(m·deg); μ_f, dynamic viscosity of the gas (fluid), kg/(m·sec); ρ, density of the gas (fluid), kg/m^3. Subscripts: e, equivalent; eff, effective; f, gas (fluid); m, tube material; s, particles; w, wall; Σ, total.

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